AJPS AMERICAN JOURNAL of POLITICAL SCIENCE

The Distortion of Related Beliefs

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Abstract: *When forming beliefs about themselves, politics, and how the world works more generally, people often face a tension between conclusions they inherently wish to reach and those which are plausible. And the likelihood of beliefs about one variable (e.g., the performance of a favored politician) depends on beliefs about other, related variables (e.g., the quality and bias of newspapers reporting on the politician). I propose a formal approach to combine these two forces, creating a tractable way to study the distortion of related beliefs. The approach unifies several central ideas from psychology (e.g., motivated reasoning, attribution) that have been applied heavily to political science. Concrete applications shed light on why successful individuals sometimes attribute their performance to luck ("imposter syndrome"), why those from advantaged groups believe they in fact face high levels of discrimination (the "persecution complex"), and why partisans disagree about the accuracy and bias of news sources.*

The world is a complicated place. When making decisions about politics (and other domains), we need to form beliefs about a wide variety of variables, such as the competence of politicians, the credibility of news sources, and the likelihood a protest will succeed. Adding to the challenge, we not only want these beliefs to be *accurate*, but also prefer to reach particular *directional* conclusions about some variables (Kruglanski 1980; Kunda 1990). This article proposes a model of belief formation that includes both accuracy and directional motives, allowing for trade-offs between these goals. These trade-offs become particularly interesting when forming beliefs about multiple variables, as the accuracy motive pushes us to reach conclusions that are jointly coherent.

Take a simple example. A newspaper reports that a politician has abused her office for private gain. A reader who likes the politician could update his beliefs about several factors. One natural factor to learn about is the quality of the politician. The fact that the news source published a critical article may also be informative about their bias. These updates are linked: If the politician really is corrupt, there is no reason to think the newspaper is biased against her, and if the newspaper is biased, one could conclude the accusations are spurious. Put another way, conditional on reading a critical article, beliefs about the performance of the politician and the bias of the newspaper become positively correlated: Higher beliefs about bias make higher beliefs about performance more plausible, and vice versa. If the reader wants to continue believing the politician is doing a good job while also maintaining a coherent worldview, he may conclude that the newspaper is biased.

I call this phenomenon the *distortion of related beliefs*. Some of our beliefs—perhaps a small fraction—are intrinsically important enough that we want to reach a certain conclusion about their value. We want to believe that we are capable and decent, that our friends and favored relatives share these traits, and that the groups we belong to are on the right side of conflicts. A much wider set of beliefs is related to those we care about, such as the accuracy of every test we have taken, whether scientific evidence backs our favored party's policy positions, or the veracity of a nasty rumor about a close friend.

To form a coherent and plausible view of the world writ large, we may distort the *auxiliary beliefs* that we do not intrinsically care about if they are related to a *core belief* over which we do have a desired conclusion. To formalize this claim, I propose a general model of belief formation that supposes people face an accuracy motive for all of their beliefs, but directional motives only apply to core beliefs.

The bulk of the article applies this idea to several concrete problems. In each, an agent observes a signal driven

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Many thanks to Abraham Aldama, Carlo Horz, Haifeng Huang, Josh Kerzer, Marko Klašnja, Marika Landau-Wells, Gabe Lenz, Marc Meredith, John Patty, Maggie Penn, Tom Pepinsky, Thomas Zeitzoff, and audience members at UC Davis, NYU, Stanford, IAST/TSE, Yale, Behavioral Models of Politics at Rice, the Alghero Political Institutions Workshop, and APSA 2018 for comments and discussion. The optimal and objectively correct conclusion is that all remaining errors are attributed to me.

American Journal of Political Science, Vol. 63, No. 3, July 2019, Pp. 675–689

 DOI: 10.1111/ajps.12435

by one factor he intrinsically cares about, and other factors with no directional motive. I use two main interpretations throughout. First, to connect with many seminal ideas and results from social psychology, the signal can represent a test of the agent's ability. Second, to illustrate the value for political applications (in addition to those flowing from the first interpretation), the signal can represent news content about the performance of a politician. To avoid juggling too much in the introduction, I primarily describe the models in terms of the first application, and then highlight the political implications.

In the first model, the signal is only a function of the agent's ability and an error term ("luck"). If the agent has directional motives to think more highly of his ability than the belief derived by Bayes' rule dictates, he can respond by upwardly distorting his self-assessment of ability, albeit at a cost to the plausibility of the view he settles on. As a by-product of this distortion, he also concludes that he was less lucky than a neutral observer would think. Conversely, if a successful agent does not want his self-assessment of ability to be too high, he may conclude that he just got lucky to distort his belief down to a more comfortable level. The latter possibility provides an explanation for the "imposter syndrome" phenomenon common among successful people (Clance and Imes 1978).

Next, suppose success is also affected by the level of discrimination faced by the agent. He now forms a joint inference about both his ability and the degree of discrimination faced by people like him (in addition to luck). Importantly, the Bayesian posterior beliefs about ability and discrimination are positively correlated; for a fixed level of success, those facing more discrimination are generally of higher ability. As a result, even if the agent does not intrinsically care about how much discrimination he faces (i.e., it is auxiliary), this belief will get distorted as well in order to reach the desired conclusion about ability while maintaining a reasonably plausible worldview. This provides an explanation for why members of objectively advantaged groups can develop a "persecution complex," believing they are the true victims of discrimination.

In the political news context, this model highlights how those with different directional motives will reach different conclusions about media bias, consistent with a large empirical literature on the "hostile media" phenomenon (starting with Vallone, Ross, and Lepper 1985; see Perloff 2015 for a recent review).

Finally, suppose the agent is also uncertain about the degree to which success is driven by ability or other factors. Those who perform well tend to believe the outcome was primarily driven by their ability (or hard work). Those who do less well are tempted to conclude the test was not accurate. However, all face a general tendency to explain their own performance (but not that of others) with other factors, as this leads to a more pliable belief about ability. That is, many claims and empirical results about attribution arise naturally from this setup (e.g., Kelley 1967; Kunda 1987; Ross 1977). The payoff of the dual interpretations here is to suggest a political analog of the fundamental attribution error: The strongest partisans (and politicians themselves) tend to be skeptical about the accuracy of all "neutral" media, and they may place more trust in news sources that are in fact inaccurate.

The primary aim of the article is synthetic. Many "nonrational" ideas about belief formation from psychology that have been applied heavily to political science and economics arise naturally when cast as a maximization problem with accuracy and directional goals. Rather than arguing any particular empirical result is better explained by this approach than existing work, my main contention is that an unusually wide swath of results spanning disciplines are all natural consequences of a simple and unified approach.

Related Models

This section briefly reviews formal models of nonstandard belief formation; discussion of theoretical and empirical work on the particular applications (e.g., motivated reasoning, discrimination, partisan interpretation of facts, attribution) is deferred until the approach is employed in each area.

Several formal models in economics and political science explore potential causes or implications of non-Bayesian formation of beliefs (e.g., Gerber and Green 1999; Levy and Razin 2015; Minozzi 2013; Ogden 2016; Ortoleva and Snowberg 2015; Patty and Weber 2007; Rabin and Schrag 1999; Stone 2017); see Bénabou and Tirole (2016) for a recent review. Even small deviations from standard Bayesian belief formation can have major implications in canonical models of political accountability (Ashworth and Bueno De Mesquita 2014; Patty and Weber 2007; Woon 2012), party competition (Nunnari and Zápal 2017; Ogden 2016), and coordination (Little 2017).

In some of this work, agents trade off material gains to hold more "pleasant" beliefs: that their job is not dangerous (Akerlof and Dickens 1982), that their investments are likely to pay off (Brunnermeier and Parker 2005), or that their accomplishments stack up well compared to others (Penn 2017). Forming incorrect beliefs about one's ability (Bénabou and Tirole 2002), valuation of goods (Heifetz and Segev 2004), or cost of fighting (Little and Zeitzoff

2017) can lead to *higher* material payoffs by solving time-inconsistency or commitment problems.

The basic innovation here is to introduce a general approach that captures the trade-off between reaching an (arbitrary) desired conclusion that is still relatively likely in the Bayesian posterior. More importantly, by treating the trade-off between accuracy and directional motives in a simple and reduced-form manner, the approach here allows for a tractable treatment of how distortions of beliefs about one variable affect beliefs about other variables. That is, rather than explaining false beliefs about different facets of the world individually, the approach proposed here allows us to model how any belief can become distorted.

The Main Idea

Here is a general model for how people form conclusions about themselves and other aspects of the world. Let $\theta = (\theta_1, \ldots, \theta_n) \in \Theta \subseteq \mathbb{R}^n$ be a vector of random variables. An agent observes a signal s, which provides information about θ. In the applications here, the signal will be unidimensional and correspond to success at a task (including a politician's performance in office).

The variables θ and s are drawn from a joint prior probability distribution $f(\theta, s)$. An actor in a standard model would form a conditional posterior belief about θ after observing s using Bayes' rule; write this as $f_{\theta|s}(\theta|s)$.

Two problems may arise for someone holding this Bayesian belief. First, the posterior belief may be a complicated object. Even when imposing a strong structure like joint normality, he must keep track of n means, n variances, and $n(n-1)/2$ covariances. Second, this posterior distribution may place heavy weight on unpleasant beliefs: that he is of low ability, that his favored political party has governed poorly, or that someone close to him has behaved improperly.

To reduce these problems, suppose the agent forms a "conclusion" about the value of θ. Intuitively, the conclusion refers to his "best guess" about the state variable θ. In doing so, he faces two motivations, which I label with the terminology from Kunda (1990). First, he would like this conclusion to be *accurate*. A natural way to model this is to assume he prefers picking conclusions that receive a relatively high likelihood or density in the Bayesian posterior.[1] Second, he may have a directional motive to reach certain conclusions.

Formally, an *optimal conclusion* $\tilde{\theta}$ is a solution to:

$$\tilde{\theta} \in \arg\max_{\theta} \log(f_{\theta|s}(\theta|s)) + v(\theta). \quad (1)$$

The $\log(f_{\theta|s}(\theta|s))$ term captures the accuracy motive. Logarithmic transformations have several desirable properties for this problem. Most importantly, if two variables are independent in the posterior belief, a logarithmic transformation ensures the overall accuracy motive is additively separable in the two variables. This transformation ensures that the conclusion about one variable can affect the optimal conclusion about the other via the accuracy motive only if they are not statistically independent. (See page 1 of the supporting information [SI] for a formal statement and further discussion.)

The v term represents the intrinsic value for holding conclusion θ. Depending on the context, several assumptions about the v term may be natural. The models here take this value function as exogenous.

An agent who cares only about accuracy is a special case of the model where the v term drops out. Such an agent picks a conclusion at the mode of the posterior distribution, analogous to maximum likelihood estimation.[2]

A natural definition for the distortion of a conclusion is how far it lies from what one with no directional motive would conclude:

Definition. The *distortion* of conclusion $\tilde{\theta}$ is

$$d(\tilde{\theta}) = \tilde{\theta} - \arg\max_{\theta} f_{\theta|s}(\theta|s).$$

At the other extreme, an agent who only cares about the directional motive is a special case in which the accuracy term drops out or is constant. The solution to Equation (1) is then to simply pick the value of θ that maximizes v independent of the signal. Here, I primarily focus on the more interesting case in which both motives matter.

What Is Going on Here. As with any formal model of belief formation or decision making, we need not believe people literally think through this optimization problem when forming conclusions. One interpretation of the optimization problem is that at the moment of forming a conclusion, the agent does carefully think through what the Bayesian belief would be and then only holds on to the

[1] This formulation is different from the probabilistic formalizations of "coherentism" as reviewed in Olsson (2017), in which the coherence of a set of beliefs is equal to the joint probability of their truth divided by either the probability of (1) at least one of them being true or (2) the product of the marginal probability of each being true.

[2] In this analogy, including the directional motive is like penalized maximum likelihood estimation.

conclusion as a summary for later use.[3] In this sense, being a "motivated reasoner" is even more computationally challenging than only following accuracy motives.

Alternatively, a frequent defense of assuming people form beliefs by Bayes' rule is that if the deviations in doing so are random (with mean zero), then they will cancel out in a large population. Of course, substantial empirical evidence indicates that modest and even major departures from this ideal are common and systematic (see Rabin 1998 for an overview). The notion of forming a conclusion used here generalizes this argument by allowing deviations from Bayesian beliefs to be biased in a predictable direction—in particular, toward beliefs that individuals want to hold for reasons outside of plausibility. This same technical approach could be used to model other motives for belief formation, such as not wanting to change one's belief from the prior; see Acharya, Blackwell, and Sen (2018) for a model of cognitive dissonance in this spirit.

Importantly, in this interpretation, we need not imagine that the agent consciously forms the Bayesian posterior and then pays a cost to deviate from it, though using language like this will be useful in describing how the calculations work. More generally, the optimization problem as specified here serves as first approximation for any process of belief formation with both accuracy and directional motives.[4]

In either case, treating belief formation as a maximization problem is more in line with "System 2" or conscious thinking, rather than a "System 1" or unconscious process (for an overview, see Lodge and Taber 2013, chap. 1). So, the model is less obviously suited to explaining phenomena like seemingly irrelevant stimuli affecting political beliefs. However, it may be useful to think of implicit attitudes, affect, and the like as factors that drive the directional motive when consciously forming beliefs.

Core and Auxiliary Beliefs. A natural way to define which beliefs "matter" for the directional motive is as follows:

Definition. θ_i is an *auxiliary variable* if v is constant in θ_i. θ_i is a *core variable* if it is not an auxiliary variable.

I refer to beliefs or conclusions about core (respectively, auxiliary) variables as core (respectively, auxiliary) beliefs or conclusions.

General Characteristics of Optimal Conclusions. An immediate consequence of the core/auxiliary definition is that the conclusion about auxiliary variable θ_i will always be the value that maximizes $f_{\theta|s}(\theta_i, \tilde{\theta}_{-i}|s)$. That is, is it the most likely value of θ_i given the signal *and the conclusion about other variables* ($\tilde{\theta}_{-i}$). If θ_i is independent of the other variables conditional on s, this is the mode of the marginal posterior distribution of θ_i. However, if θ_i is related to other beliefs, the conclusion chosen will depend on the conclusion about the state of the world writ large.

For core beliefs, there are trade-offs between these goals. To formalize, consider a more general version of Equation (1) with scale parameters $w_a > 0$ and $w_v > 0$ added to the two motives, so the maximand becomes $w_a \log f_{\theta|s}(\theta|s) + w_v v(\theta)$. Taking comparative statics on these scale parameters:

Proposition 1.

(i) The plausibility of the optimal conclusion ($f_{\theta|s}(\tilde{\theta}|s)$) is increasing in w_a and decreasing in w_v, and

(ii) the directional value associated with the optimal conclusion ($v(\tilde{\theta})$) is decreasing in w_a and increasing in w_v.

Proof. See the supporting information (page 4). ■

Naturally, when the agent cares more about the accuracy motive, he will shift to a more likely conclusion. Since the optimal conclusion requires trade-offs on the margin, this also implies that he picks a conclusion he intrinsically likes less. Conversely, as the agent cares more about the directional motive, he will pick a conclusion he intrinsically likes better at the cost of being less realistic.

If interpreting the model as describing not just what people believe but what they *say* they believe, this is consistent with empirical results that partisan differences in beliefs about political facts diminish when respondents are given monetary incentives for correct answers (Bullock, Gerber, Hill, and Huber 2015; Prior, Sood, Khanna et al. 2015).[5] More speculatively, if respondents pay a psychic cost for misreporting their

[3] See Mullainathan (2002) and Fryer, Harms, and Jackson (2013) for further discussion of this idea in other models of memory.

[4] Page 2 of the SI contains a discussion of two other potential ways to model belief formation with accuracy and directional motives (and the drawbacks of these alternatives). In one, the agent maintains a "complete" belief distribution with a penalty associated with deviations from the Bayesian posterior, and the second measures the accuracy motive as the agent trying to minimize the "error" in his conclusion.

[5] However, these studies do *not* find substantial increases in the accuracy of responses with monetary incentives. This is consistent with respondents in different parties having similar and uninformative beliefs about the questions they are asked, but different v functions.

true beliefs, then these monetary incentives could change how they process information in the first place.

We now turn to the specific applications.

Application 1: Success, Luck, and Imposter Syndrome

Consider an agent forming a conclusion about a *quality* $\theta \in \mathbb{R}$. He starts with a prior belief on θ that is normal with mean μ_θ and variance σ_θ^2. He then observes a noisy signal of the quality, given by

$$s = \theta + \epsilon, \tag{2}$$

where ϵ is normally distributed with mean 0 and variance σ_ϵ^2, independent of θ.

In this and later models, I employ two interpretations of this signal. In the first, θ is the agent's own ability on some dimension (intelligence, skill at his job, etc.). Here, a natural way to view s is a score on a test or success at a task affected by the ability in question. For this interpretation, I refer to ϵ as "luck." Call this the $\mathcal{ST}$ ("self-test") interpretation.

For the second interpretation, θ refers to the performance of a politician whom the agent is invested in supporting or opposing. Here, the signal could naturally correspond to a news story about the politician, or an opinion about the politician presented by a friend. To keep the directions of the directional motive aligned between interpretations, I primarily focus on the case in which the politician is favored by the agent. Call this the $\mathcal{PN}$ ("political news") interpretation.

The Bayesian Belief. The standard Bayesian update on θ conditional on s is normally distributed with a mean that is a weighted average of the prior and the signal:

$$\mu_\theta^B(s) \equiv \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + \sigma_\epsilon^{-2}}\mu_\theta + \frac{\sigma_\epsilon^{-2}}{\sigma_\theta^{-2} + \sigma_\epsilon^{-2}}s$$

and variance $\overline{\sigma}_\theta^2 \equiv \frac{1}{\sigma_\epsilon^{-2}+\sigma_\theta^{-2}}$. So $f_{\theta|s}(\theta|s) = \frac{1}{\overline{\sigma}_\theta}\phi(\frac{\theta-\mu_\theta^B(s)}{\overline{\sigma}_\theta})$, where ϕ is the probability density function of a standard normal random variable.

Since the mode of the Bayesian belief is the same as the mean, the distortion of the quality conclusion is $d(\tilde{\theta}) = \tilde{\theta} - \mu_\theta^B(s)$. Rearranging Equation (2), any signal and conclusion about the quality imply a conclusion about the error term: $\tilde{\epsilon} = s - \tilde{\theta}$. The conclusion about luck contains a distortion of the same magnitude, but in the opposite direction: $\tilde{\epsilon} = s - (\mu_\theta^B(s) + d(\tilde{\theta})) = s - \mu_\theta^B(s) - d(\tilde{\theta})$. Consequently, any upward distortion of the quality conclusion entails a downward distortion of the luck conclusion with equal magnitude. Conversely, a downward distortion of the quality conclusion mechanically requires an upward distortion of the conclusion about luck.

The Optimal Conclusion. The log-likelihood formulation of the accuracy motive is particularly convenient when combined with normal distributions, as the accuracy motive becomes a quadratic function centered at $\mu_\theta^B(s)$:

$$\log(f_{\theta|s}(\theta|s)) = k_1 - \frac{(\theta - \mu_\theta^B(s))^2}{2\overline{\sigma}_\theta^2}, \tag{3}$$

where k_1 collects terms that are not a function of θ and hence drops out in the maximization problem. (The subscript is to differentiate from subsequent constants.)

For now, I only assume that v is continuous and differentiable. The first order conditions for $\tilde{\theta}$ is then

$$v'(\tilde{\theta}) = \frac{\tilde{\theta} - \mu_\theta^B(s)}{\overline{\sigma}_\theta^2}. \tag{4}$$

Since the mean of the Bayesian posterior distribution is also the mode, the distortion of the belief is $d(\tilde{\theta}) = \tilde{\theta} - \mu_\theta^B(s)$. Substituting this into Equation (4) and rearranging gives an expression for the optimal distortion:

$$d(\tilde{\theta}) = v'(\tilde{\theta})\overline{\sigma}_\theta^2. \tag{5}$$

Using the $\mathcal{ST}$ interpretation, the agent will have a higher self-assessment than the Bayesian mean if and only if he prefers a higher self-assessment (on the margin). The magnitude of the distortion is increasing in the strength of the directional motive ($v'(\tilde{\theta})$) and the variance in the posterior belief about ability ($\overline{\sigma}_\theta$). The latter implies that conclusions become more distorted over characteristics the agent knows less about.

More detailed results about distortion in the agent's conclusion depends on the shape of the v function. Consider two plausible cases.

Case 1: Higher Self-Evaluation is Always Better. First, suppose the agent always wants a higher conclusion about the quality, but with diminishing marginal returns:

Proposition 2. *If v is increasing and concave, then for the optimal conclusion solving Equation* (4)*:*

(i) $\tilde{\theta} > \mu_\theta^B(s)$,
(ii) $\tilde{\theta}$ *is increasing in* s, *but*
(iii) $d(\tilde{\theta})$ *is decreasing in* s.

Proof. Parts *i-ii* follow from implicitly differentiating Equation (4). For part *iii*, consider any $s_1 < s_2$, and let $\tilde{\theta}_1$ and $\tilde{\theta}_2$ be the corresponding optimal conclusions. By part *ii* and the concavity of v, $v'(\tilde{\theta}_1) > v'(\tilde{\theta}_2)$, and, by Equation (4), $d(\tilde{\theta}_1) = \tilde{\theta}_1 - \mu_\theta^B(s_1) > \tilde{\theta}_2 - \mu_\theta^B(s_2) = d(\tilde{\theta}_2)$. ■

So the conclusion moves in the "correct" direction as the signal of quality changes, but distortion relative to the Bayesian posterior is greater when the signal is low. More on this below.

Case 2: Don't Get Too Cocky. When forming beliefs about one's ability or the performance of a favored politician, it is unreasonable to assume v is globally decreasing—that is, the agent always prefers lower conclusions. However, using interpretation $\mathcal{ST}$, suppose the agent is uncomfortable thinking his ability is "too high," either for internal reasons or to not come off as arrogant. Another plausible reason for this directional motive is that being too overconfident may lead to poor decisions. In either case, a natural way to model this premise is to assume v is a single-peaked function:

Proposition 3. *Suppose v is continuous and differentiable, and there exists a θ^* such that $v'(\theta) > 0$ for $\theta < \theta^*$ and $v'(\theta) < 0$ for $\theta > \theta^*$. Then there exists an s^* such that for $s < s^*$, the optimal conclusion solving Equation (4) is $\tilde{\theta} \in (\mu_\theta^B(s), \theta^*)$, and for $s > s^*$, $\tilde{\theta} \in (\theta^*, \mu_\theta^B(s))$.*

Proof. See the supporting information (page 5). ■

Intuitively, the agent always forms a conclusion between what he intrinsically wants to believe and what a Bayesian would think of his ability. So high performers will think they are not as good as they really are or, equivalently, think they just got lucky. Low performers will think they are better than they really are.

Summary and Empirical Discussion. Figure 1 summarizes how the conclusions about quality diverge from the Bayesian posterior mean for the two cases of the v function. In both panels, the dashed line is the 45-degree line, so conclusions further from this line represent larger distortions. The black curves correspond to a case with more uncertainty in the posterior belief (higher $\overline{\sigma}_\theta^2$), and the gray curves represent a case with less uncertainty.

The left panel illustrates the case in which higher conclusions are always better but with diminishing returns (v increasing and concave). The distortions are largest for low signals— that is, those performing poorly on the test or reading a highly negative article about the favored politician. Distortions are smaller for those who do well; eventually, the conclusion converges to the Bayesian mean. For any $\mu_\theta^B(s)$, the distortion of the conclusion is greater with more uncertainty (i.e., a higher $\overline{\sigma}_\theta^2$).

More generally, those learning unpleasant information form the most distorted beliefs. There is a pessimistic element to this result: Getting people to accept facts far from what they want to believe will always be a challenge. Still, there is a silver lining. Everyone is responsive to the information they receive, in the sense that higher signals lead to higher conclusions about whatever the signal indicates. Learning happens and "in the right direction," just not as far as a Bayesian purist would predict or hope. (See Hill 2017 for empirical evidence consistent with this prediction close to the $\mathcal{PN}$ interpretation.)

The right panel illustrates the case in which v is single peaked, and the self-assessment the agent intrinsically likes best is $\theta^* = 1$. In this case, the conclusions are above the Bayesian mean for $\mu < \theta^*$, and below for higher means.

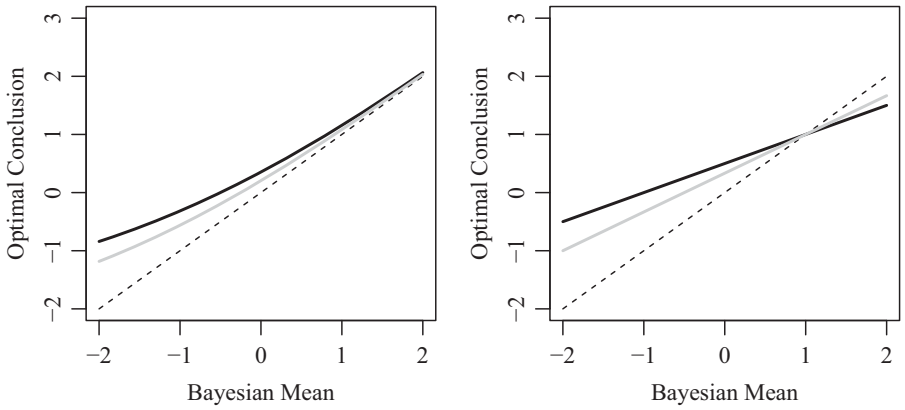
With interpretation $\mathcal{ST}$, this provides a simple theory for the origin of "imposter syndrome" among successful people (Clance and Imes 1978). Those who perform well have a high Bayesian posterior about θ and may recognize that others will interpret this to mean they are of high ability. To form a more comfortable assessment, they explain their success by ascribing it to other factors ("I just got lucky"), even if they realize others with the same data would conclude that they really have high ability.

If our agent accepts that he is of lower ability than a neutral observer would conclude, then he should expect that future signals of his performance will be lower than his past performance. So, once his conclusion is formed in this manner, it is "correct" to fear that he will be revealed as an "imposter" by future signals.

To be somewhat formal about this, suppose the agent truly has an ability $\theta = 2$. He starts with a weak prior about his ability and then observes an accurate signal $s_1 = 2$, generating a Bayesian posterior centered around $\mu_\theta^B(2) = 2$. The desire to not seem too full of himself pushes his conclusion down to $\tilde{\theta} = 1$. If he thinks that the next signal will be close to his own conclusion about ability, he will expect that the second signal will be around $s_2 = 1$. If the two signals are weighted equally, this will lead the Bayesian posterior to go down from 2 to $\mu_\theta^B(s_1, s_2) = 1.5$. However, note that his premise that s_2 will likely be around 1 is incorrect: His true ability *is* $\theta = 2$. So if the second signal is also typical, the neutral observer will be unsurprised by the agent's continued success, though he himself will just expect that the third (and later) signals will reveal him to be not as good as previously thought.

The model also suggests a connection between imposter syndrome, overconfidence, and gender. Since men are more overconfident than women in a wide variety of contexts (e.g., Ortoleva and Snowberg 2015), this connection could explain why imposter syndrome is concentrated among successful women (empirical evidence on this front is mixed, but generally in the direction that women are more apt to exhibit imposter feelings; see

FIGURE 1 Optimal Conclusions as a Function of the Bayesian Mean with Increasing and Concave (Left), and Single-Peaked (Right) v Function



Note: In both panels, the black curve represents a case with a higher posterior variance ($\overline{\sigma}^2_\theta$) than the gray curve.

Cusack, Hughes, and Nuhu 2013). In particular, suppose the overconfidence of men is driven (for whatever reason) by a stronger desire for a high self-assessment. This could be formalized by assuming men and women both have a single-peaked v function, but men tend to have a higher ideal (θ^*). If so, then (1) men will have a higher upward distortion of their conclusion about their ability, and (2) women (particularly successful ones) will have a higher upward distortion in their conclusion about how lucky they were, and a greater fear that their future performance will not live up to the past.

Application 2: Discrimination, Bias, and the "Persecution Complex"

While the model in the previous section considers the relationship between beliefs about two factors—in interpretation $\mathcal{ST}$, ability and luck—these variables are connected by a simple accounting identity. Luck was just the difference between success and ability, so increasing the conclusion about ability forced a change in the conclusion about luck. What happens if there are other factors that influence the signal?

Discrimination is one such factor. Some groups face more discrimination than others, but there can be strong disagreement about which groups are disadvantaged and to what degree. For example, substantial empirical evidence indicates that women and ethnic and religious minorities in the United States are subject to substantial discrimination in labor markets and other contexts (e.g., Riach and Rich 2002). However, a common trope on conservative media is a complaint that "if you're a Christian or a white man in the USA, it's open season on you."[6] And part of their audience agrees: In a recent survey, Evangelical Christians on average report that Christians face more discrimination in the United States than Muslims, whereas other religious groups believe the opposite.[7]

In the $\mathcal{PN}$ interpretation, the natural analog to discrimination is bias of the news source. A large literature studies the reality and perceptions of bias in news sources (e.g., Gentzkow and Shapiro 2006; Groseclose and Milyo 2005). The strand most related to the model here has shown that people generally think the media is biased against their own positions (Vallone, Ross, and Lepper 1985), particularly those who are strong partisans and highly involved in politics (Eveland and Shah 2003).

Why might such disagreements arise? To explore this question, write the signal of success as:

$$s = \theta - \delta + \epsilon,$$

where δ represents the discrimination against the agent or the news source bias against the politician. Suppose θ, δ, and ϵ are (in the prior) independent and normally distributed with means μ_θ, μ_δ, and 0, and variances σ^2_θ, σ^2_δ, and σ^2_ϵ.

[6] See http://www.wonkette.com/582723/bill-oreilly-hillary-clinton-to-murder-all-the-poor-white-christian-men-goodbye-america/.

[7] See http://www.patheos.com/blogs/godisnotarepublican/2015/07/please-stop-with-the-christian-persecution-complex-youre-embarrassing-the-faith/.

The Bayesian belief. The signal provides information about both the agent's ability and how much discrimination he faces. As derived in the SI (page 6), the joint distribution of (θ, δ) conditional on s is jointly normal with mean vector:

$$\begin{aligned}&(\mu_\theta^B(s), \mu_\delta^B(s))\\ &= \left(\frac{\mu_\theta(\sigma_\delta^2+\sigma_\epsilon^2)+(s+\mu_\delta)\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2+\sigma_\epsilon^2},\right.\\ &\qquad \left.\frac{\mu_\delta(\sigma_\theta^2+\sigma_\epsilon^2)-(s-\mu_\theta)\sigma_\delta^2}{\sigma_\theta^2+\sigma_\delta^2+\sigma_\epsilon^2}\right) \qquad (6)\end{aligned}$$

and covariance matrix

$$\begin{aligned}\overline{\Sigma} &= \begin{array}{c} \\ \theta \\ \sigma \end{array}\begin{array}{c} \begin{array}{cc}\theta & \qquad\delta\end{array} \\ \begin{pmatrix} \frac{\sigma_\delta^2\sigma_\theta^2+\sigma_\epsilon^2\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2+\sigma_\epsilon^2} & \frac{\sigma_\delta^2\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2+\sigma_\epsilon^2} \\ \frac{\sigma_\delta^2\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2+\sigma_\epsilon^2} & \frac{\sigma_\delta^2\sigma_\epsilon^2+\sigma_\delta^2\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2+\sigma_\epsilon^2} \end{pmatrix}\end{array}\\ &\equiv \begin{pmatrix} \overline{\sigma}_\theta^2 & \overline{Cov}(\theta,\delta) \\ \overline{Cov}(\theta,\delta) & \overline{\sigma}_\delta^2 \end{pmatrix}. \qquad (7)\end{aligned}$$

The individual updates resemble standard unidimensional learning models, as s is a noisy signal of θ with "error term" $\delta+\epsilon$, and also a noisy signal of $-\delta$ with "error term" $\theta+\epsilon$.

More important for our purposes, even though θ and δ were independent in the prior, *conditional on s* they have a positive covariance. This is because for a fixed degree of success, higher ability will generally be associated with facing more discrimination (e.g., "if she succeeded despite the obstacles, she must be really good," "even the liberal New Republic"). Useful for later calculations, the correlation between the two variables conditional on s is

$$\rho = \frac{\overline{Cov}(\theta,\delta)}{\overline{\sigma}_\theta\overline{\sigma}_\delta} = \frac{\sigma_\delta\sigma_\theta}{\sqrt{(\sigma_\theta^2+\sigma_\epsilon^2)(\sigma_\delta^2+\sigma_\epsilon^2)}}. \qquad (8)$$

The optimal conclusion. Suppose the belief about the quality (θ) is core, but discrimination/bias (δ) is auxiliary. The latter is not obviously so. Returning to our definition, assuming beliefs about discrimination are auxiliary implies that people do not intrinsically care about the conclusion they reach *in isolation*. For the $\mathcal{ST}$ interpretation, one may object that people really do care about their beliefs about whether people like them face discrimination. Similarly, for the $\mathcal{PN}$ interpretation, one could argue that beliefs about liberal media bias are central to conservative identity in the United States. Both objections are fair; however, the point of the modeling that follows is that these beliefs can become distorted even when considering the "hard case" in which people *do not* care about discrimination or media bias in and of itself, but because these beliefs affect their worldview more generally. Put another way, the fact that people act as if they want to hold certain beliefs about whether they face discrimination may be driven solely by the desire to protect other beliefs that are more central to their identity.

With v a function of θ but not δ, the optimal joint conclusion is[8]

$$(\tilde{\theta},\tilde{\delta}) \in \underset{(\theta,\delta)}{\arg\max}\ \log(f_{\theta,\delta|s}(\theta,\delta|s)) + v(\theta). \qquad (9)$$

The accuracy term simplifies to:

$$\begin{aligned}&\log(f_{\theta,\delta|s}(\theta,\delta|s))\\ &= k_2 - \frac{\left(\frac{(\theta-\mu_\theta^B(s))^2}{\overline{\sigma}_\theta^2} - \frac{2\rho(\theta-\mu_\theta^B(s))(\delta-\mu_\delta^B(s))}{\overline{\sigma}_\theta\overline{\sigma}_\delta} + \frac{(\delta-\mu_\delta^B(s))^2}{\overline{\sigma}_\delta^2}\right)}{2(1-\rho^2)}, \qquad (10)\end{aligned}$$

where k_2 collects the terms that do not depend on θ and δ and hence do not affect the optimization. Conveniently, Equation (10) is quadratic in both θ and δ.

Since δ only enters the accuracy term, the optimal conclusion about discrimination requires that the derivative of (10) with respect to δ is equal to zero (at $\theta=\tilde{\theta}$), which simplifies to

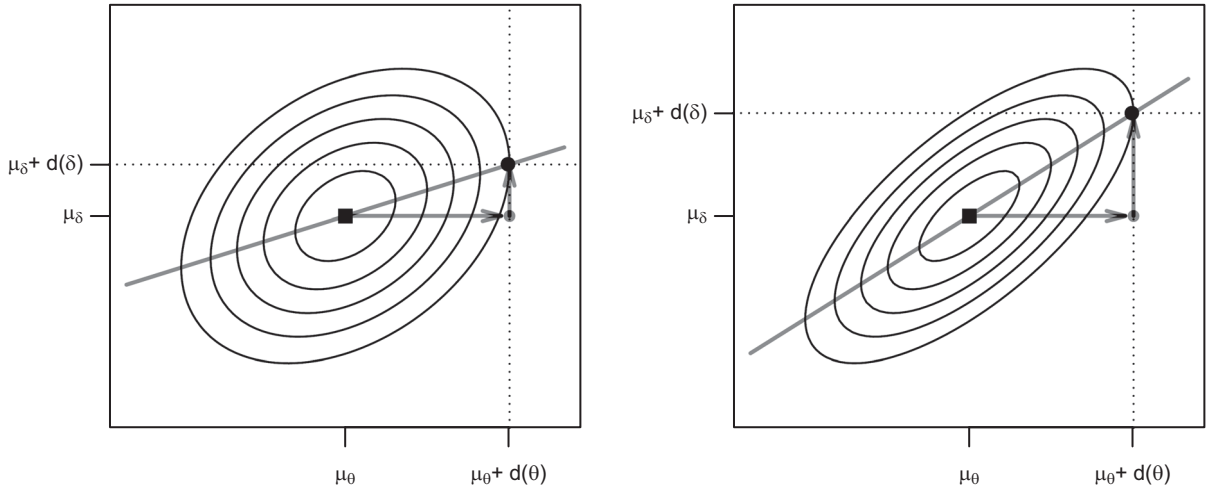
$$\begin{aligned}\tilde{\delta} &= \mu_\delta^B(s) + \tfrac{\rho\overline{\sigma}_\delta}{\overline{\sigma}_\theta}(\tilde{\theta}-\mu_\theta^B)\\ &\Leftrightarrow d(\tilde{\delta}) = \tfrac{\overline{Cov}(\theta,\delta)}{\overline{\sigma}_\theta^2}d(\tilde{\theta}). \qquad (11)\end{aligned}$$

So the distortion in the conclusion about discrimination/bias is a fraction times the distortion about the core quality θ. Further, this fraction is the ratio of the covariance between θ and δ and the variance of θ. This may look familiar as the coefficient on θ in a regression predicting δ.

Figure 2 illustrates why. Each panel plots level curves of the Bayesian posterior belief about the two variables, with higher density in curves closer to the center black square (at the mean). The gray (lower) dots are points on this posterior density when only distorting the ability belief by amount $d(\theta)$ (and $d(\delta)=0$). However, the agent can form a belief that is more plausible (at a level curve closer to the mean) by also upwardly distorting the belief about δ. For any conclusion about θ, the agent will pick the δ that maximizes the density conditional on both θ and s. Visually, this is represented by the solid points, which lie tangent to the level curves, meaning

[8] As above, this conclusion corresponds to a luck conclusion, $\tilde{\epsilon} = s-\tilde{\theta}+\tilde{\delta}$. Analogous results hold if writing the maximization problem as forming a joint inference about θ and ϵ.

FIGURE 2 The Optimal Distortion of the Belief about Discrimination as a Function of the Distortion of the Belief about Ability



Note: Each panel contains a contour plot of a posterior belief about θ and δ. In the left panel, the posterior covariance between the beliefs is 0.35, and in the right panel it is 0.7. In both panels, for a fixed distortion of θ indicated by the vertical dotted line, the optimal conclusion is at the highest-level curve of the posterior belief, which is the point along the vertical line tangent to the level curves.

higher or lower conclusions about discrimination would be less plausible (for the fixed ability conclusion). Accordingly, the ratio of these distortions is always equal to the slope of the regression line. The left panel illustrates a case in which this slope is low, and hence the distortion of the belief about discrimination is small. In the right panel, the slope is higher, and hence the discrimination belief gets distorted nearly as much as the ability belief.

Importantly, this implies that *the degree to which auxiliary beliefs get distorted is directly tied to how closely related they are to core beliefs.* With the $\mathcal{ST}$ interpretation, if discrimination does not drive much of the variance in life success, then there is little reason to distort beliefs about it. However, if believing that one faces high degrees of discrimination does make much more confident self-assessments plausible, beliefs about discrimination can be highly distorted. For the $\mathcal{PN}$ interpretation, this means that the belief about the bias of a news source will get distorted more when the reporting induces a strong correlation between the bias and performance of the politician. Revisiting Equation (8), this will tend to be true when there is little noise in the signal (σ_ϵ is small), which could be true when the news source has reported a lot on the politician in question.

To complete the derivation of the optimal assessment, plugging the optimal conclusion about δ as a function of the conclusion about θ into Equation (10) and simplifying gives the following:

$$\log\left(f_{\theta,\delta|s}\left(\theta, \mu_\delta^B(s) + \frac{\overline{Cov}(\theta,\delta)}{\overline{\sigma}_\theta^2}(\theta - \mu_\theta^B)\middle| s\right)\right)$$
$$= k_3 - \frac{(\theta - \mu_\theta^B(s))^2}{2\overline{\sigma}_\theta^2}$$

for a constant k_3. Other than this constant (which differs from k_1 in Equation (3), but also drops out when maximizing with respect to θ), this expression is the same as the log likelihood of the marginal distribution of θ. The optimal conclusion about θ (given the relationship between the optimal conclusions of θ and δ) now solves

$$v'(\tilde{\theta}) = \frac{\tilde{\theta} - \mu_\theta^B}{\overline{\sigma}_\theta^2}. \tag{12}$$

So the distortions on the belief about ability/the performance of the politician are the same as the model in the previous section, just with a different posterior variance for the belief about ability.

Summarizing:

Proposition 4. *The optimal conclusion solving Equation* (9) *is equal to the Bayesian belief plus distortions that are characterized by*

$$d(\tilde{\theta}) = v'(\tilde{\theta})\overline{\sigma}_\theta^2 \tag{13}$$

and

$$d(\tilde{\delta}) = v'(\tilde{\theta})\overline{Cov}(\theta,\delta). \tag{14}$$

Proof. This follows immediately from Equations (11) and (12). ■

This formulation highlights two factors that determine the magnitude of distortions of auxiliary beliefs: how much the agent cares about his conclusion about the core variable θ ($v'(\tilde{\theta})$), and how closely related this belief is to the auxiliary variable ($\overline{Cov}(\theta, \delta)$).

Summary and Empirical Discussion. Revisiting the motivating example, diverging views of which groups face discrimination can arise from a common desire among all individuals to think they are of high ability. The model also suggests some factors that drive beliefs about discrimination. Inspection of Equation (6) reveals that, for purely Bayesian reasons, those with a higher prior belief on their ability will tend to believe they face more discrimination for a fixed signal. On the other hand, if this prior belief is correct, those with a higher prior belief will observe higher signals (associated with less discrimination). Combining, those observing signals worse than expected will tend to believe they face more discrimination. In a dynamic setting where discrimination and luck evolve over time, this will be precisely people who had "good" draws of δ and ϵ in the past, that is, those who were previously privileged.

Further, when there are diminishing marginal returns to higher conclusions about ability (i.e., v is concave), this distortion is strongest among the unsuccessful. So we may expect to see the most distorted beliefs about discrimination among the less successful of previously privileged groups, a potentially testable hypothesis. In particular, the conclusion by white Christian males that they are held back by discrimination may be particularly alluring for those in this group who have not succeeded for other reasons (ability, luck, etc.).

More broadly, can "blaming failure on discrimination" lead to higher self-evaluations? In a sense, yes. If the presence of an indeterminate amount of discrimination makes success a noisier signal of ability, then belief distortions will be greater. But once this greater noise is accounted for, one reaches the same conclusion about ability whether jointly assessing ability and discrimination or just the former. More generally, we cannot infer from the fact that people form incorrect beliefs about auxiliary variables that this is a cause of their forming incorrect beliefs about themselves or other core variables; rather, the desire to reach a certain conclusion about the core variables is what causes the wider set of false beliefs.

With the $\mathcal{PN}$ interpretation, the model implies that those with different directional motives about the politician will reach different conclusions about the bias of the news source even if they posess the same information. Further, those with different directional motives may appear to have different "prior" beliefs even if they have the same information. For example, suppose two people with the same prior belief but different directional motives both observe the same signal. Since they have a different v function, they will reach a different conclusion. And if that conclusion acts as their prior belief (say, as measured by a researcher before giving an informational treatment) when observing a new signal, it might appear that different priors are what drive different interpretations of the second signal. However, it is really the different directional motive that led to the different prior in the first place.[9] As a result, it may prove challenging to distinguish between explanations of why different readers interpret the same new piece of information differently driven by purely Bayesian versus "behavioral" mechanisms.

Similarly, if people have prior beliefs about core variables that were influenced by directional motives, it may also be tricky to empirically distinguish between not wanting to accept unpleasant information because of current directional motives (as in the model here) or to avoid changing any belief due to confirmation bias (Rabin and Schrag 1999) or cognitive dissonance (Acharya, Blackwell, and Sen 2018). However, a recent study distinguishing between receiving new information about presidential polling that is desirable versus undesirable and confirmatory versus disconfirmatory indicates the subjects update heavily when observing disconfirmatory but desirable new information (Tappin, van der Leer, and McKay 2017). This is more consistent with the model here, where directional motives push people to favorable conclusions regardless of their prior belief.

Application 3: Attribution and News Source Quality

The final model considers a situation in which the agent is unsure what factors are most relevant in driving the signal. For the $\mathcal{ST}$ interpretation, he may make inferences not only about his ability from how well he does, but also about whether to attribute his performance to luck, skill, or other factors (Kelley 1967; Kunda 1987; Ross 1977). For the $\mathcal{PN}$ interpretation, our reader may be uncertain about how *accurate* the news source is, even setting aside issues of bias. To capture this, let the signal

[9] See Gentzkow and Shapiro (2006) for a statement of the "purely Bayesian" argument along these lines.

be

$$s = \theta + \omega\epsilon,$$

where $\omega \in \{g, b\}$, $0 < g < b$. As above, the prior on θ is normal with mean μ_θ and variance σ_θ^2. In this section, let ϵ be a standard normal random variable (i.e., with variance 1). So, the ω parameter scales how much noise the signal contains. When $\omega = g$, the signal has less noise (a "good test of ability," or an "accurate news source") compared to when $\omega = b$ (a "bad test of ability," or an "unreliable news source"). Let $\pi \in (0, 1)$ be the prior probability that the signal is good ($\omega = g$).

The agent forms his conclusion with respect to θ and ω (i.e., the quality and the degree to which the signal is driven by noise). The optimal conclusion solves

$$(\tilde{\omega}, \tilde{\theta}) \in \arg\max_{(\omega,\theta)} \log(f_{\theta,\omega|s}(\theta, \omega|s)) + v(\theta, \omega). \quad (15)$$

The Bayesian Belief. Since the agent is uncertain about ω, the posterior belief is a normal mixture:

$$f_{\theta,\omega|s}(\theta, \omega|s) = \begin{cases} Pr(\omega = g|s)\, f_{\theta|s,\omega}(\theta|s, g) & \omega = g \\ Pr(\omega = b|s)\, f_{\theta|s,\omega}(\theta|s, b) & \omega = b. \end{cases}$$

There are two pairs of terms in the density. The $f_{\theta|s,\omega}(\theta|s, \omega)$ terms are the beliefs about θ conditional on s *and* ω, which by standard analysis are normal with mean and variance:

$$\mu_\theta^B(s, \omega) = \frac{\sigma_\theta^{-2}\mu_\theta + \omega^{-1}s}{\sigma_\theta^{-2} + \omega^{-1}} \quad \text{and}$$

$$\overline{\sigma}_\theta(\omega)^2 = \frac{1}{\sigma_\theta^{-2} + \omega^{-1}}.$$

The $Pr(\omega = g|s)$ and $Pr(\omega = b|s)$ terms represent the beliefs about whether the test is good or bad given the signal. To derive these terms, conditional on ω (but not θ), the distribution of s is normal with mean μ_θ and variance $\sigma_\theta^2 + \omega^2 \equiv \sigma_s(\omega)^2$. So

$$Pr(\omega|s) = \frac{\pi \frac{1}{\sigma_s(\omega)}\phi\left(\frac{s-\mu_\theta}{\sigma_s(\omega)}\right)}{Pr(s)}.$$

(I refrain from writing out the denominator, as it drops out of relevant calculations.)

The Optimal Conclusion for a "Neutral Observer". As a benchmark, first consider the case in which both θ and ω are auxiliary. This corresponds to what the attribution literature describes as inferences made by an outside observer who does not intrinsically care about the ability of the test taker (nor the reliability of the test). In the $\mathcal{PN}$ interpretation, this could correspond to a news item about a topic for which the reader has no directional motive.

For a fixed conclusion about ω, the optimal conclusion about θ is $\mu_\theta^B(s, \omega)$. For example, once the neutral observer decides the test is accurate, he picks the most likely conclusion about the quality given $\omega = g$.

So the overall optimal conclusion is either $(g, \mu_\theta^B(s, g))$ or $(b, \mu_\theta^B(s, b))$. The good test conclusion leads to a higher posterior likelihood if and only if

$$\begin{aligned} &Pr(\omega = g|s)\, f_{\theta|s,\omega}(\mu_\theta^B(s, g)|s, g) \\ &\quad \geq Pr(\omega = b|s)\, f_{\theta|s,\omega}(\mu_\theta^B(s, b)|s, b) \\ &\frac{\pi \frac{1}{\sigma_s(g)}\phi\left(\frac{s-\mu_\theta}{\sigma_s(g)}\right)}{Pr(s)} \frac{1}{\overline{\sigma}_\theta(g)}\phi(0) \\ &\quad \geq \frac{(1-\pi)\frac{1}{\sigma_s(b)}\phi\left(\frac{s-\mu_\theta}{\sigma_s(b)}\right)}{Pr(s)} \frac{1}{\overline{\sigma}_\theta(b)}\phi(0) \\ &\frac{\pi}{1-\pi}\frac{\overline{\sigma}_\theta(b)}{\overline{\sigma}_\theta(g)} \geq \frac{\frac{1}{\sigma_s(b)}\phi\left(\frac{s-\mu_\theta}{\sigma_s(b)}\right)}{\frac{1}{\sigma_s(g)}\phi\left(\frac{s-\mu_\theta}{\sigma_s(g)}\right)}. \end{aligned} \quad (16)$$

When the two ratios on the left-hand side of Equation (16) are high, the agent tends to believe the signal is accurate. The first ratio reflects the prior information: When the prior indicates the test is likely to be accurate (high π, low $1 - \pi$), this conclusion is more likely.

Less obviously, the second ratio is the standard deviation of the posterior belief about θ with a bad test over a good test. This is always above 1, indicating a general tendency to conclude that the signal of success is accurate. Algebraically, this follows from the fact that the peaks of normal densities are higher when the standard deviation is low. The agent wants to be confident in his conclusion about θ, and believing the test had low noise allows for a more precise estimate.

If ability-as-auxiliary represents the case of assessing others, this is consistent with a key part of the fundamental attribution error (Ross 1977). If our goal when forming inferences about the ability of others is just to make them plausible, there is a bias towards thinking that outcomes are driven by ability rather than situational factors. Things will be different when ability is a core belief and the agent faces pressure to form a conclusion past the peak of the posterior density, which drops off more sharply when concluding the test is accurate.

Next, consider the right-hand side of Equation (16), which is the relative likelihood of observing s under the low- or high-noise conclusion. This will be high when s is close to μ_θ, and low when s is far from μ_θ. Intuitively, when observing a "typical" signal, the observer tends to think the test is accurate. When observing an extreme

signal, the observer becomes convinced that it must be a bad test of ability simply because the result is so unusual.

Formally:

Proposition 5. *Suppose* θ *and* ω *are both auxiliary. If* $\frac{\pi}{1-\pi}\frac{\overline{\sigma}_\theta(b)}{\overline{\sigma}_\theta(g)} \leq \frac{\sigma_s(g)}{\sigma_s(b)}$*, then the optimal conclusion solving Equation* (15) *is* $(\tilde{\omega}, \tilde{\theta}) = (b, \mu_\theta^B(s, b))$ *for all* s*. If the reverse inequality holds, then there exists a* $\underline{s}$ *and* $\overline{s} > \underline{s}$ *such that the optimal assessment is* $(\tilde{\omega}, \tilde{\theta}) = (g, \mu_\theta^B(s, g))$ *for* $s \in [\underline{s}, \overline{s}]$ *and* $(b, \mu_\theta^B(s, b))$ *for* $s \leq \underline{s}$ *and* $s \geq \overline{s}$*.*

Proof. See the supporting information (page 7). ■

A naive reading of this result would indicate that there are more circumstances in which the neutral observer believes that the signal was high noise. However, note that $\frac{\overline{\sigma}_\theta(b)}{\overline{\sigma}_\theta(g)} > 1$ and $\frac{\sigma_s(g)}{\sigma_s(b)} < 1$. So, if starting with a neutral prior on the signal's being low or high noise (i.e., $\pi = 1/2$), the agent will think the signal is primarily driven by ability for signals that are not too extreme (i.e., s close to μ_θ). For example, suppose $\sigma_\theta = g = 1$, $b = 2$, and $\pi = 1/2$. Then the chance of a signal moderate enough to induce a low-noise assessment is nearly 90%.[10] So the result is largely consistent with the idea that people tend to think the performance of others is mainly driven by their ability rather than situational factors. However, this tendency will be weaker when observing an unexpected performance level, consistent with Feather (1969).

More importantly, most of the cited results in the attribution literature are about *comparisons* between how neutral observers (for whom the ability belief is auxiliary) form conclusions versus those with a vested interest in reaching a certain conclusion (the ability belief is core). The final analysis makes this comparison.

The optimal conclusion when ability is a core belief. Now consider an agent who does care about having a high self-assessment of ability, or a reader who has a directional motive in how he or she views the subject of a news article.

To simplify, let $v(\theta) = \alpha\theta$, for $\alpha > 0$. So the agent always wants a higher conclusion about θ, and α scales the magnitude of this preference.

There are two ways that adding a directional motive affects whether the agent concludes the test is accurate. First, for any result, there is an advantage to concluding that the test is noisy, since this means there is less of a penalty for distorting the belief upward. Second, there is a tendency to want to think tests that return favorable results are accurate, since this leads to a larger increase in the mean of the Bayesian belief. As derived in the SI (page 7), the agent concludes that the test is accurate if and only if

$$\alpha\big(\mu_\theta^B(s, g) - \mu_\theta^B(s, b) + \alpha(\overline{\sigma}_\theta(g)^2 - \overline{\sigma}_\theta(b)^2)\big) \geq \log\left(\frac{\overline{\sigma}_\theta(g)\phi(\alpha\overline{\sigma}_\theta(b))Pr(\omega = b|s)}{\overline{\sigma}_\theta(b)\phi(\alpha\overline{\sigma}_\theta(g))Pr(\omega = g|s)}\right). \quad (17)$$

The left-hand side of Equation (17) represents the directional (dis)advantage of reaching the ability conclusion associated with low noise versus high noise. The right-hand side reflects the comparison between the objective likelihood of the optimal high- and low-noise conclusions.

Both sides of Equation (17) are quadratic functions in s. So, like the auxiliary case, the inequality either always holds, in which case the high-noise conclusion is always preferred, or, there is an interval of signals in which the agent thinks the test is accurate:

Proposition 6. *When* $v(\theta) = \alpha\theta$*, then there exists a* $\pi^* \in (0, 1)$ *such that:*

(i) if $\pi < \pi^*$*, then the optimal conclusion solving Equation* (15) *is* $(\tilde{\omega}, \tilde{\theta}) = (b, \mu_\theta^B(s, b) + \alpha\overline{\sigma}_\theta(b)^2)$ *for all* s*.*
If $\pi > \pi^*$*, then*
(ii) there exists a $\underline{s}$ *and* $\overline{s} > \underline{s}$ *such that the optimal conclusion is* $(\tilde{\omega}, \tilde{\theta}) = (g, \mu_\theta^B(s, g) + \alpha\overline{\sigma}_\theta(g)^2)$ *for* $s \in [\underline{s}, \overline{s}]$ *and* $(b, \mu_\theta^B(s, b) + \alpha\overline{\sigma}_\theta(b)^2)$ *for* $s \leq \underline{s}$ *and* $s \geq \overline{s}$*, where*
(iii) $\underline{s}$ *and* $\overline{s}$ *are increasing in* α*, and*
(iv) $\overline{s} - \underline{s}$ *is constant in* α*.*

Proof. See the supporting information (page 8). ■

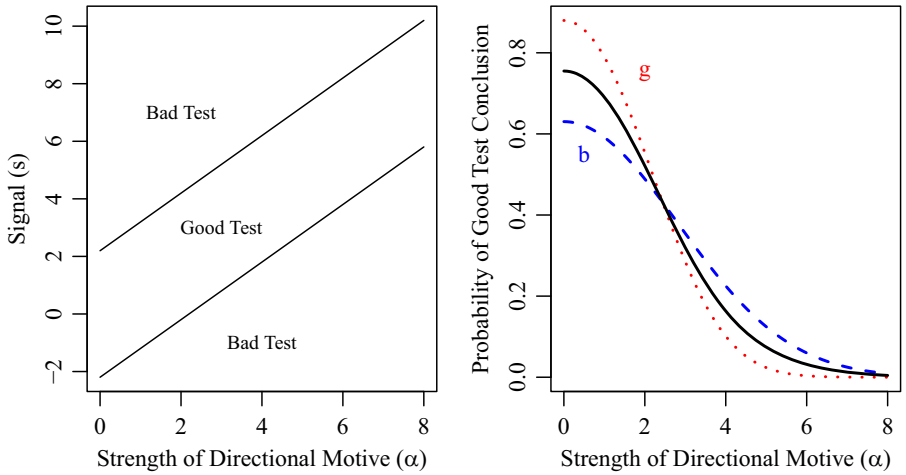
In words, unless the prior belief always forces the conclusion that the signal is noisy, then there is a window of signals during which the agent thinks the test is accurate. This window is increasing in his desire to have a high self-evaluation, though the length of the window is constant in α.

Summary and Empirical Discussion. Figure 3 shows an example of how introducing the need for positive self-evaluation affects attribution. Using the $\mathcal{ST}$ interpretation, higher values on the x-axis correspond to a greater desire to have a positive self-evaluation. For the $\mathcal{PN}$ interpretation, higher values of α correspond to a stronger desire to have a positive view of the politician.

The left panel shows which signals lead to the conclusion that the signal is a good or bad test. For signals between the two lines, the agent concludes the signal is low noise. As α increases, there is an upward shift of the window of "good test" signals. People who care more about their self-assessment of ability are more

[10] When the noise is in fact low, the probability that $s \in (\underline{s}, \overline{s})$ is 0.83, and when it is in fact high, the analogous probability is 0.9.

FIGURE 3 Range of Signals of Success Leading to Low Noise Attribution as a Function of α



apt to believe tests that are in their favor. However, no matter how much the agent wants to believe he is of high ability, extremely high signals always lead him to conclude that the test does not measure ability well.

The right panel plots the probability of a signal that leads to a low-noise conclusion as a function of α. The dotted curve shows the probability of a low-noise conclusion when the test is in fact low noise, and the dashed curve when the test is high noise. The solid curve plots the average probability of a low-noise assessment.

All three curves are decreasing in α. This is because the (unconditional) distribution of s is symmetric and single peaked around $\mu_\theta = 0$. So shifting the window of accepted signals upwards decreases the probability that the agent believes the signal is a good measure of ability. This completes the model's derivation of the fundamental attribution error: Those who care a lot about feeling high ability tend to think their performance is not primarily driven by ability, as this allows them more leeway to reach positive self-evaluations (Ross 1977).

Comparing the dotted and dashed curves, a neutral observer or someone with a lower need for a positive self-evaluation is more likely to think the test is accurate ($\omega = g$) when it is in fact accurate. Visually, the dotted curve is above the dashed curve for low α. However, the curves eventually cross. So someone who cares a great deal about a positive self-evaluation is more likely to believe inaccurate tests. Tests that are accurate generally deliver truer but less acceptable results to people with strong directional motives.

The Political Attribution Error? In the $\mathcal{PN}$ interpretation, those with strong directional motives plausibly correspond to strong partisans and those highly involved in politics, including politicians themselves. According to the model, readers without directional motives will tend to trust their sources of information, as this leads to more plausible conclusions about the subject of reporting. On the other hand, strong partisans and politicians will tend to be skeptical about the accuracy of media, which objectively is "neutral" and "accurate."[11] Further, as shown by the b curve lying above the g curve in the right panel of Figure 3, they may place more trust in news sources that are in fact *less* accurate.

What Next?

The applications in this article are wide-ranging. Empirical examples span disciplines and decades. Although it risks becoming disorienting, this broadness is purposeful, aiming to show how the approach introduced here is flexible enough to apply to many domains. What ties the results together is that they are all consequences of the maximization problem given by Equation (1), which balances the desire to reach accurate conclusions that are also intrinsically palatable, where the accuracy motive can span several variables.

In order to focus on how several prominent empirical results and observations can be cast as distorting beliefs

[11] At first glance, this may seem inconsistent with empirical results that find more partisan citizens are better informed (e.g., Palfrey and Poole 1987). However, these results are likely better explained by differential incentives to acquire information rather than how differences in partisanship affect the processing of the same information.

about one variable to reach a desired conclusion about another, I have treated the directional motive as exogenously given and avoided modeling how distorted beliefs affect decisions. To conclude, I provide some suggestions for how the model could be extended in these directions.

Microfounding the v Function. A natural way to extend the model is to endogenize the directional motive. In the context of ability, people may want to think they are of high ability to better convince others that they are capable (Trivers 2000). A similar principle could hold in the overprecision notion of overconfidence studied by Ortoleva and Snowberg (2015): Genuinely believing one's views are precise may make it easier to persuade others.

Although it does not lack empirical grounding, the directional motive driving the $\mathcal{PN}$ application—the desire to think highly of certain political leaders—has less obvious theoretical origins. One possibility is that people want to think that the groups they are a member of are good. Since partisanship can be a basis for a strong group identity and the quality of leaders reflects on the quality of the group, members may want to think highly of the leader through this channel. Another possibility is a general tendency to defer to authority, which can promote social cohesion.

The Effect on Decisions. While belief formation is a topic worthy of study by itself, most of political science (particularly formal theory) is concerned with how people make decisions given their beliefs. The model of belief formation proposed here could be dropped into nearly any incomplete information model.

A general class of problems for which this model could prove fruitful is in studying information acquisition. For example, what types of news sources would someone with accuracy and directional motives seek out? And how would those decisions affect media organizations' incentives to provide certain kinds of news?

Another possible direction is to study how voters processing information in this manner would affect politician's behavior. For example, do directional motives undermine politician's incentives to work on behalf of constituents? And how does this question interact with the way directional motives affect media behavior? The results about beliefs becoming more distorted over uncertain variables also may have implications for how precise politicians want to be when speaking.

A Final Thought. The notion that formal theories of politics must involve selfish actors maximizing their material gains given correctly formed beliefs is long dead, and good riddance. However, most deviations from this paradigm have involved more general assumptions about preferences, such as adding altruism, an expressive/"warm glow" payoff for participation, or loss aversion. Nonstandard treatment of beliefs has been less common. This may be partly driven by the fact that fiddling with utility functions requires no changes to standard solution concepts, which tell us how to translate any set of utility functions (and other assumptions about the environment) to behavioral predictions. When changing assumptions about beliefs, things are harder: In addition to figuring out which deviations from using Bayes' rule to formalize, the modeler must also face challenges in determining how these distorted beliefs map to actions, and, in a game-theoretic setting, how higher-order beliefs map to actions. Should actor A know that actor B forms incorrect beliefs? Does B know that A knows he forms incorrect beliefs, and if so, why doesn't A correct his beliefs?

The model here does not answer these questions, but hopefully providing a simple and tractable formulation of how to model distorted beliefs in a multivariate environment will be a useful first step in building applied models with more general and realistic belief formation.

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Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Data S1